

# NAG C Library Function Document

## nag\_prob\_studentized\_range (g01emc)

### 1 Purpose

nag\_prob\_studentized\_range (g01emc) returns the probability associated with the lower tail of the distribution of the Studentized range statistic.

### 2 Specification

```
double nag_prob_studentized_range (double q, double v, Integer ir, NagError *fail)
```

### 3 Description

The externally Studentized range,  $q$ , for a sample,  $x_1, x_2, \dots, x_r$ , is defined as:

$$q = \frac{\max(x_i) - \min(x_i)}{\hat{\sigma}_e},$$

where  $\hat{\sigma}_e$  is an independent estimate of the standard error of the  $x_i$ 's. The most common use of this statistic is in the testing of means from a balanced design. In this case for a set of group means,  $\bar{T}_1, \bar{T}_2, \dots, \bar{T}_r$ , the Studentized range statistic is defined to be the difference between the largest and smallest means,  $\bar{T}_{\text{largest}}$  and  $\bar{T}_{\text{smallest}}$ , divided by the square root of the mean-square experimental error,  $MS_{\text{error}}$ , over the number of observations in each group,  $n$ , i.e.,

$$q = \frac{\bar{T}_{\text{largest}} - \bar{T}_{\text{smallest}}}{\sqrt{MS_{\text{error}}/n}}.$$

The Studentized range statistic can be used as part of a multiple comparisons procedure such as the Newman–Keuls procedure or Duncan's multiple range test (see Montgomery (1984) and Winer (1970)).

For a Studentized range statistic the probability integral,  $P(q; v, r)$ , for  $v$  degrees of freedom and  $r$  groups can be written as:

$$P(q; v, r) = C \int_0^\infty x^{v-1} e^{-vx^2/2} \left\{ r \int_{-\infty}^\infty \phi(y) [\Phi(y) - \Phi(y - qx)]^{r-1} dy \right\} dx,$$

where

$$C = \frac{v^{v/2}}{\Gamma(v/2) 2^{v/2-1}}, \quad \phi(y) = \frac{1}{\sqrt{2\pi}} e^{-y^2/2} \quad \text{and} \quad \Phi(y) = \int_{-\infty}^y \phi(t) dt.$$

The above two-dimensional integral is evaluated using numerical quadrature with the upper and lower limits computed to give stated accuracy (see Section 7).

If the degrees of freedom  $v$  are greater than 2000 the probability integral can be approximated by its asymptotic form:

$$P(q; r) = r \int_{-\infty}^\infty \phi(y) [\Phi(y) - \Phi(y - q)]^{r-1} dy.$$

This integral is evaluated using nag\_1d\_quad\_inf (d01amc).

### 4 References

Abramowitz M and Stegun I A (1972) *Handbook of Mathematical Functions* (3rd Edition) Dover Publications

Lund R E and Lund J R (1983) Algorithm AS 190: probabilities and upper quartiles for the studentized range *Appl. Statist.* **32** (2) 204–210

Montgomery D C (1984) *Design and Analysis of Experiments* Wiley

Winer B J (1970) *Statistical Principles in Experimental Design* McGraw–Hill

## 5 Parameters

1: **q** – double *Input*  
*On entry:* the Studentized range statistic,  $q$ .  
*Constraint:*  $q > 0.0$ .

2: **v** – double *Input*  
*On entry:* the number of degrees of freedom for the experimental error,  $v$ .  
*Constraint:*  $v \geq 1.0$ .

3: **ir** – Integer *Input*  
*On entry:* the number of groups,  $r$ .  
*Constraint:*  $ir \geq 2$ .

4: **fail** – NagError \* *Input/Output*  
 The NAG error parameter (see the Essential Introduction).

## 6 Error Indicators and Warnings

### NE\_INT

On entry,  $ir = \langle value \rangle$ .  
 Constraint:  $ir \geq 2$ .

### NE\_ACCURACY

Warning - There is some doubt as to whether full accuracy has been achieved.

### NE\_REAL

On entry,  $q = \langle value \rangle$ .  
 Constraint:  $q > 0.0$ .  
 On entry,  $v = \langle value \rangle$ .  
 Constraint:  $v \geq 1.0$ .

### NE\_BAD\_PARAM

On entry, parameter  $\langle value \rangle$  had an illegal value.

### NE\_INTERNAL\_ERROR

An internal error has occurred in this function. Check the function call and any array sizes. If the call is correct then please consult NAG for assistance.

## 7 Accuracy

The returned value will have absolute accuracy to at least four decimal places (usually five), unless **fail.code** = **NE\_ACCURACY**. When **fail.code** = **NE\_ACCURACY** it is usual that the returned value will be a good estimate of the true value.

## 8 Further Comments

None.

## 9 Example

The lower tail probabilities for the distribution of the Studentized range statistic are computed and printed for a range of values of  $q$ ,  $\nu$  and  $r$ .

### 9.1 Program Text

```
/* nag_prob_studentized_range (g01emc) Example Program.
*
* Copyright 2001 Numerical Algorithms Group.
*
* Mark 7, 2001.
*/
#include <stdio.h>
#include <nag.h>
#include <nag_stdlib.h>
#include <nagg01.h>

int main(void)
{
    /* Scalars */
    double q, v, valp;
    Integer exit_status, i, ifail, ir;
    NagError fail;

    Vprintf("g01emc Example Program Results\n");
    /* Skip heading in data file */
    Vscanf("%*[^\n] ");

    INIT_FAIL(fail);
    exit_status = 0;
    Vprintf("\n%s\n", "q      v      ir      Quantile ");
    for (i = 1; i <= 3; ++i)
    {
        Vscanf("%lf%lf%ld*[^\n] ", &q, &v, &ir);
        ifail = -1;
        valp = g01emc(q, v, ir, &fail);

        if (fail.code == NE_NOERROR)
        {
            Vprintf("%7.4f%2s%4.1f%1s%3ld%1s%10.4f\n", q, "", v, "", ir, "", valp);
        }
        else
        {
            Vprintf("Error from g01emc.\n%s\n", fail.message);
            exit_status = 1;
        }
    }
    return exit_status;
}
```

### 9.2 Program Data

```
g01emc Example Program Data
4.6543 10.0 5
2.8099 60.0 12
4.2636 5.0 4
```

### 9.3 Program Results

g01emc Example Program Results

q	v	ir	Quantile
4.6543	10.0	5	0.9500
2.8099	60.0	12	0.3000
4.2636	5.0	4	0.9000

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